國立高雄師範大學 100 學年度學士班轉學生招生考試試題

系所別：數學系三年級

科　目：高等微積分（全一頁）

※注意：1. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上，於本試題上作答者，不予計分。

2. 限用藍色或黑色之鋼筆、原子筆作答，除製圖外，以鉛筆或其他顏色作答者不予計分。

1. (a) $\phi \neq K \subset R^n$, (b) $p \notin K$ is an accumulation point of K. Show that there exists a sequence $\{z_n\}_{n=1}^{\infty}$ in K such that $\lim_{n\to\infty} z_n = p$ （25%）

2. Define $<\square,\square>: R^n \to R$ by $< x, y >= \sum_{k=1}^{n} x_k y_k$ ， $\forall x = (x_1, x_2, ..., x_n); y = (y_1, y_2, ..., y_n) \in R^n$. Show that the function defined above is an inner product for R^n （25%）

3. Prove that $f(x) = \frac{1}{x}$ is not uniformly continuous on $S = (0,1]$ （20%）

4. Suppose that the sequence $\{f_n\}$ of functions converges uniformly to f on S. Assume that each function f_n is continuous at $c \in S$. Prove that the limit function f is also continuous at c. （10%）

5. Given the following real-valued function

$$f(x,y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0), \end{cases}$$

Find $\frac{\partial f}{\partial x \partial y}(0,0)$ and $\frac{\partial f}{\partial y \partial x}(0,0)$. （20%）