國立高雄師範大學 102 學年度學士班轉學生招生考試試題

系所別：數學系三年級

科　目：高等微積分　（全一頁）

※注意：1.不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上，於本試題上作答者，不予計分。

2.限用藍色或黑色之鋼筆、原子筆作答，以鉛筆或其他顏色作答者不予計分。

Let R be the set of all real numbers, $[a,b]$ be a closed and bounded interval in R, $cl(A)$ be the closure of set A, and $C([a,b],R)$ be the set of all continuous functions $f:[a,b]\to R$.

1. Let A and B be nonempty subsets of R. Assume that A and B are bounded above. Prove that $\sup(A+B)=\sup(A)+\sup(B)$.（10%）
2. Let (M,d) be a metric space and $B,A\subseteq M$. Prove that if A is connected and $A\subseteq B\subseteq cl(A)$, then B is connected.（10%）
3. Prove or disprove the following statements:
   (a) Let (M,d) and (N,ρ) be two metric spaces, and $f:M\to N$ be a continuous function. If $A\subseteq M$ is compact, then $f(A)$ is compact.（10%）
   (b) Let $f:R\to R$ be a continuous function. Then the zero set $\{x\in R: f(x)=0\}$ of f is compact.（10%）
   (c) Let (M,d) be a metric space and $f,g:M\to R$ are continuous on M. Then $A=\{x\in M: f(x)>g(x)\}$ is open in M and $B=\{x\in M: f(x)\ge g(x)\}$ is closed in M.（10%）
4. Let $f:R\to R$ be differentiable. Assume that for all $x\in R,\ 0\le f'(x)\le f(x)$. Prove that if $f(z)=0$ for some $z\in R$, then $f(x)=0$ for all $x\in R$.（10%）
5. Prove that the limit $\lim_{n\to\infty}\int_0^3\sqrt{\sin\left(\frac{x}{n}\right)+x+1}\,dx$ exists and evaluate it.（10%）
6. (a) Prove that $\sum_{n=1}^{\infty}\frac{(-1)^n e^{-nx}}{n}$ converges uniformly on $[0,1]$.（10%）

   (b) Show that $\lim_{n\to\infty}\int_0^{\infty}e^{-nx}\cdot\frac{\sin x}{x}\,dx=0$.（10%）
7. Show that $\sum_{k=0}^{\infty}\frac{(-1)^k}{2k+1}=\frac{\pi}{4}$.（10%）