

國立高雄師範大學 104 學年度學士班轉學生招生考試試題

系所別：數學系三年級

科目：高等微積分（全一頁）

※注意：1.不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上，於本試題上作答者，不予計分。

2.限用藍色或黑色之鋼筆、原子筆作答，以鉛筆或其他顏色作答者不予計分。

1. Let $A \subseteq \mathbb{R}^n, u \in A$, and consider the mapping

$$F = (F_1, \dots, F_m): A \rightarrow \mathbb{R}^m.$$

Prove that $F: A \rightarrow \mathbb{R}^m$ is continuous at $u \Leftrightarrow \forall F_i: A \rightarrow \mathbb{R}$ is continuous at u . (20%)

2. Let $f: \mathbb{R}^n \rightarrow \mathbb{R}^n, \exists \alpha \in [0,1) \ni \|f(x) - f(y)\| \leq \alpha \|x - y\|, \forall x, y \in \mathbb{R}^n$. Prove that

$$\exists w \in \mathbb{R}^n \ni f(w) = w. \quad (20\%)$$

3. Prove that $\left| \sum_{k=1}^{\infty} \left(1 - \cos \frac{1}{k}\right) \right| \leq 2$. (20%)

4. Let $f \in C^\infty(a,b)$. Prove that f is analytic on (a,b) if and only if f' is analytic on (a,b) . (20%)

5. Compute the iterated limits at $(0,0)$ of each of the following functions. Determine which of these functions has a limit as $(x,y) \rightarrow (0,0)$ in \mathbb{R}^2 , and prove that the limit exists:

$$(1) f(x,y) = \frac{\sin x \sin y}{x^2 + y^2}. \quad (10\%)$$

$$(2) f(x,y) = \frac{x-y}{(x^2 + y^2)^\alpha}, \quad \alpha < \frac{1}{2}. \quad (10\%)$$