

國立高雄師範大學 105 學年度學士班轉學生招生考試試題

系所別：數學系三年級

科目：高等微積分

※注意：1.不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上，於本試題上作答者，不予計分。

2.限用藍色或黑色之鋼筆、原子筆作答，以鉛筆或其他顏色作答者不予計分。

1. Let $f : [a, b] \rightarrow \mathbb{R}$ be a function. f is said to be Hölder's continuous on $[a, b]$ if there exist nonnegative real constants C, α s.t.,

$$|f(x) - f(y)| \leq C |x - y|^\alpha \quad \text{for all } x, y \in [a, b].$$

If f is continuously differentiable on $[a, b]$, prove that f is Hölder's continuous on $[a, b]$. (10%)

2. Show that $f(x) = \frac{\sin x}{x}$ is conditionally integrable on $[1, \infty)$. (10%)

3. Decide which of the following functions are improperly integrable on I : (10%)

(a) $f(x) = \frac{\log^a x}{x^p}$ where $a > 0$ is fixed and $p > 1$; $I = (1, \infty)$.

(b) $f(x) = \log(\sin x)$; $I = (0, 1)$.

4. Let $A, B \subset \mathbb{R}^n$. Define $d(A, B) = \inf\{\|x - y\| : x \in A, y \in B\}$. If A is compact, B is closed and $A \cap B = \emptyset$ prove $d(A, B) > 0$. (10%)

5. Let $F(y) = \int_{\sin y}^{\cos y} x e^{x^2 + y^2} dx$. Find $F'(y)$. (20%)

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6. Find the following limit. (20%)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy + y^3}{x^2 + y^2}$$

7. Let $f(x, y) = \begin{cases} \frac{x^2 - xy}{x + y} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$. Find $\nabla f(0, 0)$ (10%)

8. Find the following limit. (10%)

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \ln \sqrt[n]{1 + \frac{k}{n}}$$