

# 國立高雄師範大學 106 學年度學士班轉學生招生考試試題

系所別：數學系 數學組、應數組 三年級

科 目：高等微積分

※注意：1. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上，於本試題上作答者，不予計分。

2. 限用藍色或黑色之鋼筆、原子筆作答，以鉛筆或其他顏色作答者不予計分。

1. (a) Let  $S$  be a metric space and  $A_1, A_2, \dots, A_n$  be a finite family of closed sets in  $S$ . Show that the union of  $A_1, A_2, \dots, A_n$  is a closed set. (10%)

(b) Give an example to show that the union of infinitely many closed sets may not be closed. (5%)

2. Determine the improper integral  $\int_0^{\pi} \frac{\sin x}{x\sqrt{x}} dx$  converges or diverges. (10%)

3. Let  $\{f_n\}$  be a sequence of real-valued continuous functions defined on  $[0, 1]$  such that  $f_n \rightarrow f$  uniformly. Prove or disprove that  $\lim_{n \rightarrow \infty} \int_0^{1-\frac{1}{n^2}} f_n(x) dx = \int_0^1 f(x) dx$  (15%)

4. Find the Fourier series of  $f(x) = x \sin x, -\pi \leq x \leq \pi$ . (10%)

5. Evaluate  $\lim_{n \rightarrow \infty} \int_0^1 \frac{\sqrt{x}-1}{\ln x} x^n dx$  (10%)

(背面有題 續翻背面)

系所別：數學系 數學組、應數組 三年級

科 目：高等微積分

6. (a) Let  $f: R \rightarrow R$  be continuous. Is  $f^{-1}(K)$  compact for every compact subset  $K$  of  $R$ . (10%)
- (b) Let  $f: R \rightarrow R$  be continuous. If  $f^{-1}(K)$  is compact for every compact subset  $K$  of  $R$ , is  $f$  continuous on  $R$ ? (10%)
7. Let  $\{f_n\}$  be a sequence of continuously differentiable functions on  $[a, b]$  such that  $f_n(a) = f_n(b) = 0$  and  $\int_a^b |f'_n(x)|^2 dx \leq M$  for some fixed constant  $M$ . Prove that  $\{f_n\}$  contains a subsequence that converges uniformly on  $[a, b]$ . (20%)