

國立高雄師範大學 108 學年度學士班轉學生招生考試試題

系所別：數學系三年級

科 目：高等微積分

※注意：1.不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上，於本試題上作答者，不予計分。

2.限用藍色或黑色筆作答，以其他顏色作答者不予計分。

1. (15%) Show that the following limits exist and evaluate them :

$$(a) \lim_{n \rightarrow \infty} \int_0^3 \sqrt{\sin\left(\frac{x}{n}\right) + x + 1} dx$$

$$(b) \lim_{n \rightarrow \infty} \int_0^1 \frac{\sin(nx)}{nx} dx$$

$$(c) \lim_{n \rightarrow \infty} \int_0^{\infty} e^{-nx} \cdot \frac{\sin x}{x} dx$$

2. (15%) Let $f_n: [0,1] \rightarrow \mathbb{R}$ be a integrable function, $n \in \mathbb{N}$ and $f_n \rightarrow f$ uniformly on $[0,1]$. Show that if $b_n \nearrow 1$ as $n \rightarrow \infty$, then

$$\lim_{n \rightarrow \infty} \int_0^{b_n} f_n(x) dx = \int_0^1 f(x) dx$$

3. (20%) Show that

$$f(x) = \sum_{k=1}^{\infty} \frac{1}{k} \sin\left(\frac{x}{k+1}\right)$$

converges, pointwise on \mathbb{R} and uniformly on each bounded interval in \mathbb{R} , to a differentiable function f which satisfies

$$|f(x)| \leq |x| \quad \text{and} \quad |f'(x)| \leq 1 \quad \text{for all } x \in \mathbb{R}$$

4. (20%) Evaluate the improper integral $\int_0^{\infty} e^{-x^2} dx$.

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5. (15%) Given $f(x_1, \dots, x_n) = x_1^2 + \dots + x_n^2$, find the directional derivative $f'(c; u)$ at $c = (c_1, \dots, c_n)$ in the direction $u = (u_1, \dots, u_n)$.

6. (15%) Prove $\ln(1-x) = -\sum_{k=1}^{\infty} \frac{x^k}{k}$ for $-1 < x < 1$.
(Hint: $\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$ for $-1 < x < 1$)