

國立高雄師範大學 103 學年度學士班轉學生招生考試試題

系所別：數學、物理、光電與通訊工程、電子工程等學系二年級

科目：微積分

※注意：1.不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上，於本試題上作答者，不予計分。

2.限用藍色或黑色之鋼筆、原子筆作答，以鉛筆或其他顏色作答者不予計分。

1. Evaluate (a) $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$ (5%) (b) $\int \ln x dx$ (5%) (c) $\int_{-2}^2 |x^2 - x - 2| dx$ (5%)

2. Prove by Mean Value Theorem that $5 + \frac{5}{52} < \sqrt{26} < 5 + \frac{1}{10}$ (10%)

3. Solve $\lim_{n \rightarrow 1} \frac{(1-\sqrt{x})(1-\sqrt[3]{x}) \cdots (1-\sqrt[n]{x})}{(1-x)^{n-1}} = ?$ (5%)

4. Evaluate (a) $\int_1^{\infty} \frac{dx}{x + x\sqrt{2}}$ (5%) (b) $\frac{d}{dx} \int_{\cos x}^{\sin x} \sqrt{1-t^2} dt$ (5%)

5. Determine the convergence of (a) $\sum_{n=1}^{\infty} \sin \frac{1}{n}$ (5%) (b) $\sum_{n=1}^{\infty} \tan^{-1} \left(\frac{n}{\sqrt{n}} \right)$ (5%)

6. Find the values of the following problems :

(a) $\frac{d}{dx} \left(x \int_2^{x^2} \sin(t^3) dt \right) = \underline{\hspace{2cm}}$ (10%)

(b) $\lim_{x \rightarrow \infty} (1 + \sin x)^{1/x} = \underline{\hspace{2cm}}$ (10%)

(c) $\lim_{k \rightarrow \infty} \left(1 + \frac{r}{k} \right)^k = \underline{\hspace{2cm}}$ for any $r \in \mathbb{R}$. (10%)

(背面有題 續翻背面)

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7. Evaluate the integral $\int \tan^2 x \sec x \, dx$. (10%)

8. Evaluate the double integral $\int_{1/y}^2 \int_{\sqrt{y/x}}^y \sqrt{\frac{y}{x}} e^{\sqrt{xy}} \, dx \, dy$ by applying the transformation:

$u = \sqrt{xy}$ and $v = \sqrt{y/x}$ and integrating over an appropriate region in the uv -plane.
(10%)