

# 國立高雄師範大學 105 學年度學士班轉學生招生考試試題

系所別：數學、物理、光電與通訊工程、軟體工程與管理等學系二年級

科目：微積分

※注意：1.不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上，於本試題上作答者，不予計分。

2.限用藍色或黑色之鋼筆、原子筆作答，以鉛筆或其他顏色作答者不予計分。

1. Find  $\lim_{x \rightarrow 2} \frac{2x(x-2)}{|x-2|}$  (5%)

2. Find  $\frac{dy}{dx}$  of the following problems

(1)  $y = (x+1)^x$  (5%)

(2)  $x^3 - xy - y^3 = 1$  (5%)

3. Find  $f'(0)$  if  $f\left(\frac{x-1}{x+1}\right) = 2x$  (5%)

4. Let  $f(x) = \begin{cases} x \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$  (You have to justify your answers.)

(1) Is it continuous at  $x = 0$ ? (5%)

(2) Is it differentiable at  $x = 0$ ? (5%)

5. Determine the local extreme values for the following function: (10%)

$$f(x) = \begin{cases} 3-x, & \text{if } x < 0 \\ 3+2x-x^2 & \text{if } x \geq 0 \end{cases}$$

6. Let  $f(x) = x^3 - 3x^2 - 1$  and  $x > 2$

(1) Show that  $f(x)$  is one-to-one function (5%)

(2) Let  $f^{-1}$  be the inverse function. Find the value of  $\left. \frac{df^{-1}(t)}{dt} \right|_{t=-1}$  (5%)

(背面有題 續翻背面)

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7. Evaluate the integrals :

$$(1) \int_{-\pi/4}^{\pi/4} [(\sin t)\mathbf{i} + (1 + \cos t)\mathbf{j} + (\sec^2 t)\mathbf{k}] dt \quad (5\%)$$

$$(2) \int (\sin 2x - \csc^2 x) dx \quad (5\%)$$

$$(3) \int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2} dy dx \quad (5\%)$$

8. Find the lengths of the following curves :

$$(1) r = \sqrt{1 + \sin 2\theta}, \quad 0 \leq \theta \leq \pi\sqrt{2} \quad (5\%)$$

$$(2) r = 1 + \cos \theta \quad 0 \leq \theta \leq \pi \quad (5\%)$$

$$(3) x = \int_0^y \sqrt{\sec^4 t - 1} dt, \quad -\frac{\pi}{4} \leq y \leq \frac{\pi}{4} \quad (5\%)$$

9. Use the transformation  $x = u + \frac{1}{2}v$ ,  $y = v$  to evaluate the integral

$$\int_0^2 \int_{y/2}^{y/4} y^3 (2x - y) e^{-x-y^2} dx dy$$

by first writing it as an integral over a region  $G$  in the  $uv$ -plane. (20%)