

國立高雄師範大學 107 學年度學士班轉學生招生考試試題

系所別：數學系、物理學系、光電與通訊工程學系二年級

科 目：微積分

※注意：1.不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上，於本試題上作答者，不予計分。

2.限用藍色或黑色之鋼筆、原子筆作答，以鉛筆或其他顏色作答者不予計分。

1. Evaluate the limits :

(1) $\lim_{x \rightarrow e^+} (\ln x)^{1/(x-e)}$ (6%)

(2) $\lim_{x \rightarrow \infty} \left(\frac{1}{x} \int_x^{x^2+1} e^{t^2} dt \right)$ (6%)

2. Determine whether or not the given vector field is conservative. If it is conservative, find a function f such that $F = \nabla f$. (12%)

$$F(x, y, z) = xy^2z^3 \mathbf{i} + 2x^2yz^3 \mathbf{j} + 3x^2y^2z^2 \mathbf{k}.$$

3. Use the transformation $x = u + \frac{1}{2}v$, $y = v$ to evaluate the integral

$$\int_0^2 \int_{y/2}^{(y+4)/2} y^3 (2x-y) e^{(2x-y)^2} dx dy$$

by first writing it as an integral over a region G in the uy -plane. (13%)

4. Find the plane tangent to the surface $z = x^2 + \frac{1}{4}y^2$ at the point $(-1, 0, 1)$. (12%)

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5. Consider a curve Γ which is defined as the graph of $f(x) = x^2 + 2x$ in \mathbb{R}^2 . Find the length of Γ between points $(-1, -1)$ and $(0,0)$. (13%)

6. Evaluate

$$\iint_{\Omega} \frac{y^2}{x^2 + y^2} dA$$

where $\Omega = \{(x,y) \in \mathbb{R}^2 | 1 \leq x^2 + y^2 \leq 4\}$ (13%)

7. Use ratio test to determine whether $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ is convergent. (12%)

8. Interchange the order of integration and evaluate it (in any order you wish):

$$\int_1^2 \int_0^{\sqrt{4-y^2}} 2y \, dx dy.$$

(13%)