

# 國立高雄師範大學九十八學年度轉學生招生考試試題

系所別：數學、光通、電子、軟工等系二年級（以鉛筆作答者不予計分）

科目：微積分（第一頁，共二頁）

※注意：不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上，於本試題上作答者，不予計分。

壹、填空題：（每題 4%；不需要寫出演算過程）

1. What is the slope of the tangent line to the graph of  $xy^2 - 3x^2y + 2 = 0$  at the point  $(1, 1)$ .  
\_\_\_\_\_
2. Let  $f(x) = \frac{x(x-1)(x-2)(x-3)}{\cos x}$ . Find  $f'(0) = ?$  \_\_\_\_\_
3. Let  $f(x) = \frac{(x+1)^2}{1+x^2}$ . What is the maximum value of  $f(x)$ ? \_\_\_\_\_
4. If  $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} f'(x) = \lim_{x \rightarrow 0} f''(x) = \dots = \lim_{x \rightarrow 0} f^{(2009)}(x) = 0$  and  $\lim_{x \rightarrow 0} \frac{xf^{(2010)}(x)}{f^{(2009)}(x)} = 10$ , then  
 $\lim_{x \rightarrow 0} \frac{xf'(x)}{f(x)} = ?$  \_\_\_\_\_
5. Find the area of the region bounded by the graphs of  $f(x) = \sqrt{x}$  and  $g(x) = -x$  on the interval  $[0, 1]$ . \_\_\_\_\_
6.  $\int_0^1 x^2 e^x dx = ?$  \_\_\_\_\_
7.  $\int_0^1 \int_y^1 \frac{\sin x}{x} dx dy = ?$  \_\_\_\_\_
8. What is the local minimum value of the function  $f(x, y) = xy + x^{-1} + 8y^{-1}$ ? \_\_\_\_\_

（背面有題 續翻背面）

第一頁、共二頁

系所別：數學、光通、電子、軟工等系二年級（以鉛筆作答者不予計分）

科目：微積分（第二頁，共二頁）

貳、計算及證明題：（需要寫出演算或證明過程，只寫答案不予計分）

1. Show that  $\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$ ,  $x \in R$ . (10%)

2. The region bounded by the curve  $y = \sqrt{x}$ , the  $x$ -axis, and the line  $x = 4$  is revolved about the  $x$ -axis to generate a solid. Find the volume of the solid by two different methods (Disk method and Shell method). (8%)

3. Evaluate the given integrals. (30%)

(a)  $\int \frac{1}{\sqrt{4x^2 + 4x + 7}} dx$

(b)  $\int \frac{1}{1+e^x} dx$

(c)  $\int_0^1 x\sqrt{1+x} dx$

(d)  $\lim_{n \rightarrow \infty} \int_0^1 \frac{nx^{n-1}}{1+x} dx$

(e)  $\iint_R \sqrt{x^2 + y^2} dA$ , where  $R = \{(x, y) : (x-1)^2 + y^2 \leq 1\}$ .

4. Let  $f(x) = \int_0^x e^{-t^2} dt$ . Find  $f^{(7)}(0)$ . (10%)

5. Let  $g(x)$  be the inverse function of  $f(x) = x \ln x$  for  $x \geq 1$ . Find  $\int_0^e g(x) dx$ . (10%)