國立高雄師範大學 100 學年度學士班轉學生招生考試試題

系所別：數學系二年級

科　目：線性代數（第一頁，共二頁）

※注意：1. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上，於本試題上作答者，不予計分。
2. 限用藍色或黑色之鋼筆、原子筆作答，除製圖外，以鉛筆或其他顏色作答者不予計分。

◈ Denote by □ and □ the set of real numbers and the set of complex numbers, respectively.

1. (a) If V and W are two vector spaces with $\dim(V) = \dim(W)$, is $V = W$ always true? **Justify your answer**.（5%）

   (b) If $A, B, C \in M_n(□)$ with $A \neq O_n$ and $AB = AC$, is $B = C$ always true? **Justify your answer**.（5%）

2. Let $A=\{(1,1,0),(1,0,1),(0,1,1)\}$ and $B=\{(1,1,0),(1,0,1),(0,1,1),(1,1,1)\}$. Show that $\mathrm{span}(A) = \mathrm{span}(B) = □^3$.（10%）

3. Denote by $P_n(□)$ the set of polynomials with coefficients from □ and having degree less than or equal n. Let $T: P_2(□) \rightarrow P_3(□)$ be defined by

$$T(f(x)) = xf(x) + f'(x).$$

   (a) Show that T is linear.（5%）

   (b) Is T one-to-one?（7%）　　**(Justify your answer!)**

   (c) Find a basis for $R(T)$.（8%）

4. Let $A, B \in M_{m\times n}(F)$, where $F =$ □ or □. If A is row equivalent to B, show that there exists a nonsingular matrix $P \in M_m(F)$, such that $B = PA$.（10%）

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5. Let $A=\begin{pmatrix}1 & 2 & 3\\ 1 & 0 & 1\\ 1 & 1 & -1\end{pmatrix}$ and $S:\begin{cases}x_1+2x_2+3x_3=2\\ x_1\qquad\quad + x_3=3\\ x_1+x_2-x_3=1\end{cases}$. （25%）

(a) Find A^{-1}.

(b) Use A^{-1} to solve the system S.

(c) Use Cramer's Rule to solve the system S.

6. Let $A=\begin{pmatrix}1 & -1\\ 1 & 3\end{pmatrix}$. （25%）

(a) Find the characteristic polynomial $f(t)$ of A.

(b) Show that $f(A)=0$ (0 is the zero 2×2 matrix).

(c) Is A diagonalizable? Why?

(d) Find the Jordon form of A.