

國立高雄師範大學 101 學年度學士班轉學生招生考試試題

系所別：數學系二、三年級

科 目：線性代數

※注意：1.不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上，於本試題上作答者，不予計分。

2.限用藍色或黑色之鋼筆、原子筆作答，以鉛筆或其他顏色作答者不予計分。

1. Let $A = \begin{pmatrix} 3 & 2 & 3 & -2 & 1 \\ 1 & 1 & 1 & 0 & 3 \\ 1 & 2 & 1 & -1 & 2 \end{pmatrix}$. Find $\text{rank}(A)$ and the set of solutions of systems

$$\begin{cases} 3x_1 + 2x_2 + 3x_3 - 2x_4 = 1 \\ x_1 + x_2 + x_3 = 3. \\ x_1 + 2x_2 + x_3 - x_4 = 2 \end{cases} \quad (15\%)$$

2. Evaluate the determinant of $\begin{pmatrix} 0 & 2 & 1 & 3 \\ 1 & 0 & -2 & 2 \\ 3 & -1 & 0 & 1 \\ -1 & 1 & 2 & 0 \end{pmatrix}$ (10%)

3. $S = \{a_1 = (1,1,c), a_2 = (1,-1,2), a_3 = (-1,1,1)\}$.

(a) Find c such that S forms an orthogonal basis for \mathbb{R}^3 . (5%)

(b) Express $(2,4,4)$ as a linear combination of a_1, a_2, a_3 . (10%)

4. Is $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ diagonalizable? Why? (10%)

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5. Let A be a $5 \times n$ matrix of rank r and let \vec{b} be a vector in \mathbb{R}^5 . For each pair of values of r and n , indicate whether the system $A\vec{x} = \vec{b}$ is consistent or not and how many solutions it has.

(a) $n=6, r=5$ (5%)

(b) $n=6, r=4$ (5%)

(c) $n=4, r=4$ (5%)

6. Let $A = (\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4, \vec{a}_5)$ be a 4×5 matrix with column vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$, and \vec{a}_5 . Let

$A\vec{x} = \vec{b}$ be a linear system whose augmented matrix $(A|\vec{b})$ has the reduced row echelon form

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & 2 & | \\ 0 & 0 & 0 & 1 & 2 & | \\ 0 & 0 & 0 & 0 & 0 & | \\ 0 & 0 & 0 & 0 & 0 & | \end{array} \right]$$

(a) Find all solutions to the system. (10%)

(b) If $\vec{a}_1 = \begin{bmatrix} -1 \\ 1 \\ 2 \\ 5 \end{bmatrix}$ and $\vec{a}_4 = \begin{bmatrix} 3 \\ 2 \\ 4 \\ -1 \end{bmatrix}$, determine \vec{b} . (10%)

7. Let A be a particular 2×2 matrix. Determine whether the following are subspaces of the space V of 2×2 real matrices. Give your reasoning.

(a) $S_1 = \{B \in V \mid AB \neq BA\}$; (5%)

(b) $S_2 = \{B \in V \mid AB - B = 0\}$. (10%)