國立高雄師範大學 102 學年度學士班轉學生招生考試試題

系所別：數學系二、三年級

科　目：線性代數　（全一頁）

※注意：1.不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上，於本試題上作答者，不予計分。
2.限用藍色或黑色之鋼筆、原子筆作答，以鉛筆或其他顏色作答者不予計分。

1. Suppose that A is an $n \times n$ matrix with the property that $\mathbf{A}^2 = \mathbf{A}$.
   (a) Show that if λ is an eigenvalue of A, then $\lambda = 0$ or $\lambda = 1$.（10%）
   (b) Prove that A is diagonalizable.（10%）

2. Prove or give a counterexample:
   Let A and B be two $n \times n$ matrices.
   (a) If $\mathbf{B}^2$ is similar to $\mathbf{A}^2$, then B is similar to A.（10%）
   (b) If B is similar to A, and A is symmetric, then B is symmetric.（10%）
   (c) If B is similar to A, then rank(B)=rank(A).（10%）

3. Let T be a function from C^3 into C^3 defined by
   $T(x_1, x_2, x_3) = (x_1 + x_2, 2x_1 + 3x_2 + 2x_3, -x_1 - 2x_2 - 2x_3)$.
   (a) Show that T is a linear transformation.（10%）
   (b) What are the conditions on a, b, and c that (a,b,c) be in the range of T? What is the rank of T?（10%）
   (c) What are the conditions on a, b, and c that (a,b,c) be in the null space of T? What is the nullity of T?（10%）

4. Let V be the vector space of all polynomials from R into R of degree 2 or less, i.e., the space of all functions f of the form $f(x) = c_0 + c_1 x + c_2 x^2$. Let t be a fixed real number and define $g_1(x) = 1, g_2(x) = x + t, g_3(x) = (x+t)^2$.
   (a) Prove that $B = \{g_1, g_2, g_3\}$ is a basis for V.（10%）
   (b) If $f(x) = c_0 + c_1 x + c_2 x^2$, what are the coordinates of f in this ordered basis.（10%）