國立高雄師範大學 105 學年度學士班轉學生招生考試試題

系所別：數學系二、三年級

科　目：線性代數（全一頁）

※注意：1.不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上，於本試題上作答者，不予計分。
2.限用藍色或黑色之鋼筆、原子筆作答，以鉛筆或其他顏色作答者不予計分。

1. Let V and W be vector spaces over a field F with zero vectors θ_V and θ_w, respectively. Suppose that $\{v_1, v_2, \cdots, v_n\}$ is a basis for V and $T: V \to W$ is linear and one-to-one.
   (a) Show that $\{T(v_1), T(v_2), \cdots, T(v_n)\}$ is linearly independent.（5%）
   (b) Show that R(T) = span($\{T(v_1), T(v_2), \cdots, T(v_n)\}$).（5%）
   (c) If dim$W$ = n, show that T is onto.（10%）

2. Let $A \in \mathrm{M}_{n\times n}(\mathbf{R})$. If A is invertible, show that $A^{-1} = \dfrac{1}{\det(A)} adj(A)$.（20%）

3. Suppose $T: R^2 \to R^2$ is a linear transformation and that $T(1,1) = (1,4), T(1,0) = (2,5)$.
   (a) What is $T(3,5)$?（4%）
   (b) What is the kernel of T?（4%）
   (c) What is the image of T?（4%）
   (d) Is T one to one? Explain your answer.（4%）
   (e) Is T onto? Explain your answer.（4%）

4. Let $P_2 = \{a_0 + a_1 x + a_2 x^2 \mid a_0, a_1, a_2 \in R\}$. Suppose $T: P_2 \to P_2$ is linear and defined by $T(ax^2 + bx + c) = cx^2 + bx + a$.
   Find a basis β of P_2 such that $[T]_\beta$ is a diagonal matrix.（20%）

5. If U and W are subspaces of vector space V and $\dim(U) = 2$, show that either $U \subseteq W$ or $\dim(U \cap W) \le 1$.（20%）