國立高雄師範大學九十八學年度轉學生招生考試試題

系所別：數學系二年級　　　　　　　　（以鉛筆作答者不予計分）

科　目：線性代數（第一頁，共二頁）

※注意：不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上，於本試題上作答者，不予計分。

◈ Denote by $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{C}$ the set of positive integers, the set of real numbers and the set of complex numbers, respectively.

1. Let S be a vector space over $\mathbb{R}$ spanned by the following three linearly independent vectors

   $x=\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}, y=\begin{pmatrix}1\\2\\0\\-1\end{pmatrix}, z=\begin{pmatrix}3\\1\\1\\-1\end{pmatrix}$. Find an orthonormal basis for the vector space S. (10%)

2. Find all the solutions for the system. (10%)

$$\begin{aligned} x_1+2x_2+2x_3 &= 0\\ 2x_1+5x_2+7x_3 &= 0\\ 3x_1+6x_2+6x_3 &= 0 \end{aligned}$$

3. For $A=\begin{bmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{4} & \frac{3}{4}\end{bmatrix}$, evaluate $\lim_{n\to\infty} A^n$. (10%)

4. Determine if the matrix

   $A=\begin{bmatrix}-1 & -1 & -2\\ 8 & -11 & -8\\ -10 & 11 & 7\end{bmatrix}$ is diagonalizable. (Justify your answer) (10%)

5. Let T be a linear operator on an inner product space V, and suppose that $\|T(x)\|=\|x\|$ for all x in V. Is the linear operator T one-to-one ? (Justify your answer). (10%)

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6. Let A and B be subspaces of a vector space V. State whether the following is also a subspace of V. Prove or give a counter example.
   (a) $A \cap B$.　(4%)
   (b) $A \cup B$.　(4%)
   (c) $(V \setminus A) \cap B$.　(4%)

7. Let $V=\{(a,b):a,b \in \mathbb{R}\}$. For any $r \in \mathbb{R}$ and any $(a,b), (c,d) \in V$, we define
   $$(a,b)+(c,d)=(a+c,b+d)$$
   and
   $$r(a,b)=\begin{cases}(0,0) & \text{if } r=0 \\ \left(ra, \frac{b}{r}\right) & \text{if } r \neq 0\end{cases}$$
   Is V a vector space over $\mathbb{R}$ with these operations? Justify your answer.　(6%)

8. Let V be a vector space over a field F. If W_1 and W_2 are subspaces of V. Show that $\dim(W_1+W_2)=\dim(W_1)+\dim(W_2)-\dim(W_1 \cap W_2)$.　(10%)

9. Let $W=\{(a_1, a_2, a_3, a_4, a_5) \in \mathbb{R}^5: a_1+a_2+a_3+a_4+a_5=0\}$.
   (a) Show that W is a subspace of $\mathbb{R}^5$ over $\mathbb{R}$.　(5%)
   (b) Find a basis of W.　(5%)
   (c) Find dim(W).　(2%)

10. Let $L=\{A \in M_{8x8}(\mathbb{R}): A^T=A\}$.
   (a) Show that L is a vector space.　(5%)
   (b) Find dim(L).　(5%)