

國立高雄師範大學 111 學年度學士班轉學生招生考試試題

系所別：電機工程學系 二年級

科目：微積分（全一頁）

- ※注意：1.不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上，於本試題上作答者，不予計分。
2.限用藍色或黑色筆作答，以其他顏色作答者不予計分。

◆計算題（需詳列計算過程）每大題 10 分

1. (a) Find an equation of the tangent line to the curve $y = \sqrt[4]{x} - 2x$ at the given point $(1, -1)$. (5%)

(b) Sketch the graph of the first derivative $h'(x)$ where $h(x) = |x - 1| + |x + 2|$. (5%)

2. Evaluate the integrals.

(a) $\int_0^{\pi} \cos^2 \theta d\theta$. (5%) (b) $\int_{-\pi/2}^0 \cos(x) \sin(x) dx$. (5%)

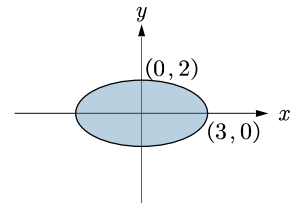


Fig. 1.

3. Show the area enclosed by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ in Fig. 1 is 6π . (10%)

4. Let $f(x) = \frac{3(x^2+2x-1)}{2x^3+x^2-x}$. Decompose the function into $f(x) = \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{x+1}$.

(a) Find the values of A, B and C . (3%)

(b) Evaluate the indefinite integral $\int f(x) dx$. (7%)

5. (a) Prove that the inequality $\left| \int_a^b f(x) dx \right| \leq \int_a^b |f(x)| dx$ holds for any function $f(x)$ that is continuous on $[a, b]$. (5%)

(b) Show that $\left| \int_a^b f(x) \cos x dx \right| \leq \int_a^b |f(x)| dx$. (5%)

6. Evaluate $\lim_{x \rightarrow 1^+} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$. (10%)

7. Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$. (10%)

8. The region \mathcal{R} enclosed by the curves $y = x$ and $y = x^2$ is rotated about the x -axis. Find the volume of the resulting solid. (10%)

9. The curve $y = \sqrt{4 - x^2}$, $-1 \leq x \leq 1$, is an arc of the circle $x^2 + y^2 = 4$. Find the area of the surface obtained by rotating this arc about the x -axis. (10%)

10. Find an equation of the plane that passes through the points $P(1, 3, 2)$, $Q(3, -1, 6)$, and $R(5, 2, 0)$. (10%)