國立高雄師範大學 114 學年度碩士班招生考試試題

系所別：數學系

科　目：基礎數學

※注意：1.作答時請將試題題號及答案依序寫在答案卷上，於本試題上作答者，不予計分。
2.答案卷限用藍、黑色筆作答，以其他顏色作答之部分，該題不予計分。

$\mathbb{R}$ denotes the set of real numbers.

1. (10%) Find $F'(x)$ if

$$F(x) = \int_0^{x^{\sqrt{x}}+cos(5-tan\,x)+log_{10}(e^{secx})} sin\,t^4\,dt, 0 < x < \frac{\pi}{2}.$$

2. (10%) Consider the sequence

$$a_1 = 4, \quad a_{n+1} = \sqrt{(6+a_n)}, \ \ n = 2,3, \ldots$$

Show that the sequence is convergent and find it's limit.

3. (10%) Find the volume of the solid formed by revolving the unbounded region lying between the graph of $f(x) = 1/x$ and the x-axis $(x \geq 1)$ about x-axis, and show that this solid has an infinite surface area.

4. (10%) Let

$$f(x) = \begin{cases} \dfrac{sin\,x^2}{x}, & x \neq 0, \\ A, & x = 0. \end{cases}$$

Find the value of A make f continuous at $x = 0$, and then, calculate $f'(x)$ and show that $f'(x)$ is continuous at $x = 0$.

5. (10%) Evaluate

$$\int_0^1 \int_{\sqrt{y}}^1 y\,cos(x^5 - 1)\,dxdy.$$

（背面尚有試題）

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6. Let $M = \begin{pmatrix} 2 & 0 & 2 & 5 \\ 0 & 2 & 1 & 4 \\ a & b & c & d \\ -2 & 4 & 0 & 3 \end{pmatrix}$, where $a, b, c, d \in \mathbb{R}$.

(1)(5%) Calculate the determinant of M, denoted as $\det(M)$.

(2)(5%) Provide values of a, b, c, d such that $\mathrm{rank}(M) = 3$.

7. Let $V = \mathbb{R}[x]$ denote the vector space of all polynomials with real coefficients.

(1)(5%) Show that the set $S = \{a + bx + cx^2 \mid a, b, c \in \mathbb{R}\}$ forms a subspace of V.

(2)(5%) Let $T: S \to S$ be defined as $T(s) = ((1 + 2x) \cdot s)'$ for $s \in S$, where $(\cdot)'$ represents the formal derivative of polynomials. Prove that T is a linear transformation.

(3)(5%) Find a basis for S and determine the corresponding matrix A for T.

(4)(8%) Compute the eigenvalues of A from (3) and find their corresponding eigenspaces.

(5)(7%) Determine if there exists an invertible matrix Q such that $Q^{-1}AQ$ is diagonal. If it exists, provide such a matrix Q.

8. (10%) Let U and W be subspaces of a vector space V. Suppose $U \cup W = V$. Show that either $U = V$ or $W = V$.