國立高雄師範大學 111 學年度碩士班招生考試試題

系所別：電機工程學系

科　目：基礎工程數學（微分方程、線性代數）

※注意：1.作答時請將試題題號及答案依序寫在答案卷上，於本試題上作答者，不予計分。
2.答案卷限用藍、黑色筆作答，以其他顏色作答之部分，該題不予計分。

1. Find the solutions of the following ordinary differential equations (ODE):

(a) $\frac{dy}{dx} = 2x + 2,\ y(0) = 1$. (5%)

(b) $\frac{dy}{dx} = (1 - 2x + 3x^2)y,\ y(1) = 2$ (5%)

2. Consider the second order ODE $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x = 3e^{-t}$ subject $x(0) = 2$ and $x'(0) = 3$.

Determine:

(a) The homogeneous solution $x_h(t)$. (3%)

(b) The particular solution $x_p(t)$. (5%)

(c) The general solution $x(t)$. (5%)

3. Solve the ODE $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 4x = 0$ subject to $x(0) = 1$ and $x\left(\frac{\pi}{2}\right) = e^{-\frac{\pi}{2}}$. (8%)

4. Consider the second order ODE $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 6t + 11$ subject $x(0) = 1$ and $x'(0) = 2$.

Determine:

(a) The homogeneous solution $x_h(t)$. (3%)

(b) The particular solution $x_p(t)$. (5%)

(c) The general solution $x(t)$. (5%)

5. Give an example of a linear system modeled by a second order ODE. (6%)

6. Solve the following system. (6%)

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 14 \\ 2x_1 + 5x_2 + 8x_3 = 36 \\ x_1 - 2x_2 = -3 \end{cases}$$

7. For the matrix $\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 1 & -2 & 0 \end{bmatrix}$, compute (a) $\det(\boldsymbol{A})$ (b) $N(\boldsymbol{A})$ (c) $R(\boldsymbol{A})$ (d) $\boldsymbol{A}^{-1}$. (16%)

8. Suppose $\boldsymbol{A}=\begin{bmatrix}1 & 0 & 0\\ 1 & 2 & -2\\ 1 & 4 & -4\end{bmatrix}$.

(a) Find the eigenvalues and the corresponding eigenvectors of $\boldsymbol{A}$. (8%)

(b) Factor $\boldsymbol{A}$ into a product $\boldsymbol{XDX}^{-1}$ where $\boldsymbol{D}$ is a diagonal matrix. (6%)

9. Let $\boldsymbol{A}=\begin{bmatrix}1 & 1 & 2\\ 0 & 2 & -1\\ 1 & -1 & 0\\ 1 & 0 & -2\end{bmatrix}$.

(a) Show that the columns of $\boldsymbol{A}$ are mutually orthogonal. (5%)

(b) Given $\overline{\boldsymbol{b}}=\begin{bmatrix}0\\ 1\\ 2\\ 3\end{bmatrix}$, find the vector $\hat{\boldsymbol{x}}$, such that $\left\|\overline{\boldsymbol{b}}-\boldsymbol{A}\hat{\boldsymbol{x}}\right\|$ is minimum. (9%)