國立高雄師範大學 115 學年度碩士班招生考試試題

系所別：電機工程學系

科　目：工程數學（線性代數與微分方程）

※注意：1.作答時請將試題題號及答案依序寫在答案卷上，於本試題上作答者，不予計分。
2.答案卷限用藍、黑色筆作答，以其他顏色作答之部分，該題不予計分。

PART I. Multiple Choice Questions (10%)（單選題，只需填寫答案不用計算過程）

1. Let $\mathbf{a}=(1,2,3)$ and $\mathbf{b}=(2,-1,1)$. Which of the following statements is correct?

(A) $\boldsymbol{a}\cdot\boldsymbol{b}=0$, therefore $\boldsymbol{a}$ and $\boldsymbol{b}$ are orthogonal.
(B) $\boldsymbol{a}\times\boldsymbol{b}$ is a scalar equal to $|\boldsymbol{a}||\boldsymbol{b}|\sin\theta$.
(C) $\boldsymbol{a}\cdot\boldsymbol{b}=|\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, which represents the projection magnitude of $\boldsymbol{a}$ onto $\boldsymbol{b}$.
(D) $|\boldsymbol{a}\times\boldsymbol{b}|=|\boldsymbol{a}||\boldsymbol{b}|cos\theta$.

2. Let $T: R^n \to R^m$ be a matrix transformation defined by $T(\mathbf{x})=A\mathbf{x}$, where A is an $m\times n$ matrix. Which of the following statements is **always true**?

(A) $T(\mathbf{x}_1+\mathbf{x}_2)=T(\mathbf{x}_1)+T(\mathbf{x}_2),\ \mathbf{x}_1,\mathbf{x}_2\in\mathbb{R}^n$.
(B) $T(\mathbf{x})$ preserves the length of every vector $\mathbf{x}\in R^n$.
(C) T is one-to-one if and only if $m=n$.
(D) T is onto if and only if the columns of A are linearly independent.

3. Let A be a complex square matrix. Which of the following statements is **always true**?

(A) If A is Hermitian, then all eigenvalues of A are real.
(B) If A is Unitary, then $A=A^*$.
(C) If A is Normal, then A must be Hermitian.
(D) If A is Hermitian, then $A^*A=I$.

4. Let $A=\begin{bmatrix}1&0\\0&2\\0&0\end{bmatrix}$. Find the **singular values** of the matrix A.

(A) $2,1$ (B) $\sqrt{5},0$ (C) $4,1$ (D) $2,1,0$ (E) None of above

5. Which of the following statements is **true**?

(A) A set of vectors is linearly independent if at least one vector in the set is nonzero.
(B) A set of vectors $\mathbf{v}_1,\mathbf{v}_2,\ldots,\mathbf{v}_k$ is linearly independent if $c_1\mathbf{v}_1+c_2\mathbf{v}_2+\cdots+c_k\mathbf{v}_k=0$ implies $c_1=c_2=\cdots=c_k=0$
(C) If a set of vectors spans $\mathbb{R}^n$, then the set must be linearly independent.
(D) If the number of vectors exceeds the dimension of the space, then the set is linearly independent.

（背面尚有試題）

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PART II. Calculation Problems (90%)（計算題，此大題均需寫出計算過程）

1. A point (x', y') is rotated counterclockwise by an angle θ to (x, y), where

$$x = x'\cos\theta - y'\sin\theta$$
$$y = x'\sin\theta + y'\cos\theta$$

(a) Express the vector $[x\ y]$ in terms of $[x'\ y']$ using matrix notation. **(3%)**

(b) Use Gauss–Jordan elimination to solve for x' and y' in terms of x and y.
【本小題限用 Gauss–Jordan 消去法】 **(4%)**

(c) Based on (a), use Cramer's rule to solve for x' and y' in terms of x and y.
【本小題限用 Cramer's rule】 **(3%)**

2. Consider the following homogeneous system of linear equations:

$$\begin{cases} x_1 + 2x_2 - x_3 + x_5 + 3x_6 = 0, \\ 2x_1 + 4x_2 - 2x_3 + x_4 + 3x_5 + 7x_6 = 0, \\ x_1 + 2x_2 - x_3 + x_4 + 2x_5 + 4x_6 = 0, \\ 3x_1 + 6x_2 - 3x_3 + x_4 + 4x_5 + 8x_6 = 0. \end{cases}$$

(a) Find the rank and nullity of the coefficient matrix. **(3%)**

(b) Solve the system and write the general solution in terms of parameters. **(4%)**

(c) Find a basis for the solution space (null space). **(3%)**

3. Given the vectors in $\mathbb{R}^3$

$$\mathbf{v}_1 = (1, 1, 0), \quad \mathbf{v}_2 = (1, 0, 1), \quad \mathbf{v}_3 = (0, 1, 1),$$

use the **Gram–Schmidt process** to find an **orthonormal basis** for $\operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. **(10%)**

4. Consider the quadratic form written by

$$P = x_1^2 + 5x_2^2 - 4x_3^2 + 6x_1x_2 - 8x_1x_3 + 10x_2x_3.$$ **(10%)**

(a) Express P in the matrix notation $\mathbf{x}^T A \mathbf{x}$, where A is **symmetric**.

(b) Find an **orthogonal change of variable** that eliminates the cross-product terms in the quadratic form, and express P in terms of the new variables.

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5. Solve $y''' + y'' = e^x \cos x$. **(10%)**

6. Solve $x^3 y''' - 6y = 0$. **(10%)**

7. Find the general solution of the following nonhomogeneous differential equations. **(15%)**

$$y'' - y = \frac{2e^x}{e^x + e^{-x}}$$

8. A series circuit contains an inductor, a resistor, and a capacitor for which $L = 0.5$H, $R = 10$ Ω, and $C = 0.01$ F, respectively. The voltage

$$E(t) = \begin{cases} 10, & 0 \le t < 5 \\ 0, & t \ge 5 \end{cases}$$

is applied to the circuit. Determine the instantaneous charge $q(t)$ on the capacitor for $t > 0$ if $q(0) = 0$ and $q'(0) = 0$. **(15%)**